# 52-nd Mathematical Olympiad in Poland
## Second Round, February 23–24, 2001

### First Day

**1.** Let $k, n > 1$ be positive integers such that the number $p = 2k - 1$ is a prime. Prove that, if the number

$$\binom{n}{2} - \binom{k}{2}$$

is divisible by $p$, then it is divisible by $p^2$.

**2.** Three points $A, B, C$ lie in this order on a line and satisfy the inequality $AB < BC$. The points $D, E$ are the vertices of the square $ABDE$. The circle with diameter $AC$ intersects the line $DE$ in the points $P, Q$ with $P$ belonging to the segment $DE$. The lines $AQ$ and $BD$ intersect in the point $R$. Prove that $DP = DR$.

**3.** Let $n \geq 3$ be a positive integer. Prove that any polynomial of the form

$$x^n + a_{n-3}x^{n-3} + a_{n-4}x^{n-4} + \ldots + a_1x + a_0$$

where at least one of the real coefficients $a_0, a_1, \ldots, a_{n-3}$ is not equal to zero, possesses less than $n$ real roots (the roots are counted with their multiplicities).

### Second Day

**4.** Find all positive integers $n \geq 3$ for which the following statement is true: any arithmetic progression $a_1, a_2, \ldots, a_n$ of the length $n$ for which the number $1a_1 + 2a_2 + \ldots + na_n$ is rational contains at least one rational value.

**5.** The point $I$ is the incenter of the triangle $ABC$. The line $AI$ intersects the side $BC$ in the point $D$. Prove that $AI + CD = AC$ if and only if $\sphericalangle B = 60^\circ + \frac{1}{3}\sphericalangle C$.

**6.** Let $n$ be a positive integer and denote by $S_n$ the set $\{1, 2, \ldots, 2n\}$. Moreover let us define two families of subsets of the set $X$:
$A_n = \{A \subset S_n : |A| = n$ and the sum of the elements of the set $A$ is even$\}$,
$B_n = \{B \subset S_n : |B| = n$ and the sum of the elements of the set $B$ is odd$\}$.
For all $n$ find $|A_n| - |B_n|$.
($|X|$ denotes the number of the elements of the set $X$).