1. For any real number \( a \) determine the number of the ordered triples \((x, y, z)\) of real numbers satisfying the following system of equations

\[
\begin{align*}
    x + y^2 + z^2 &= a \\
x^2 + y + z^2 &= a \\
x^2 + y^2 + z &= a 
\end{align*}
\]

2. Point \( P \) lies inside triangle \( ABC \) and satisfies the conditions:

\[
\angle PBA = \angle PCA = \frac{1}{3}(\angle ABC + \angle ACB).
\]

Prove that

\[
\frac{AC}{AB + PC} = \frac{AB}{AC + PB}.
\]

3. Given is a set of \( n \) points \((n \geq 2)\); no three of the points are collinear. We colour all the line segments with endpoints in this set so that two segments with a common endpoint are of different colours. Determine the least number of colours, for which there exists such a colouring.

4. Find all triples of positive integers with the following property: The product of any two of these numbers gives the remainder 1 upon division by the third number.

5. We have thrown \( k \) white dice and \( m \) black dice. Find the probability that the remainder upon division by 7 of the number on the faces of the white dice is equal to the remainder upon division by 7 of the number on the faces of the black dice.

6. In a cube of the edge 1, eight points are given. Prove that two of the points are the endpoints of a segment of length not greater than 1.