First Day

1. Let $A_n = 1, 2, \ldots, n$. Prove or disprove the following statement:
   for all integers $n \geq 2$ there exist functions $f : A_n \to A_n$ and $g : A_n \to A_n$
   which satisfy
   
   \[ f(f(k)) = g(g(k)) = k \text{ for } k = 1, 2, \ldots, n; \]
   
   \[ g(f(k)) = k + 1 \text{ for } k = 1, 2, \ldots, n - 1. \]

2. In triangle $ABC$ the angle $\angle BCA$ is obtuse and $\angle BAC = 2 \angle ABC$. The line
   through $B$ and perpendicular to $BC$ intersects line $AC$ in $D$. Let $M$ be the
   midpoint of $AB$. Prove that $\angle AMC = \angle BMD$.

3. a) Assume that nonnegative numbers $a, b, c, d, e, f$ with sum equal to 1 satisfy
   
   \[ ace + bdf \geq \frac{1}{108}. \]

   Show that
   
   \[ abc + bcd + cde + def + efa + fab \leq \frac{1}{36}. \]

   b) Do there exist six different positive numbers $a, b, c, d, e, f$ with sum equal to 1 for which
   the two above inequalities become equalities?

Second Day

4. Find all pairs $(x, y)$ of integers which satisfy the equation $x^2 + 3y^2 = 1998x$.

5. Suppose that nonnegative numbers $a_1, a_2, \ldots, a_7, b_1, b_2, \ldots, b_7$ satisfy
   
   \[ a_i + b_i \leq 2 \text{ for } i = 1, 2, \ldots, 7. \]

   Prove that there exist two different indices $k, m \in \{1, 2, \ldots, 7\}$ for which
   
   \[ |a_k - a_m| + |b_k - b_m| \leq 1. \]

6. Prove that in tetrahedron $ABCD$ the edge $AB$ is perpendicular to the edge $CD$ if and only if there exists a parallelogram $CDPQ$ such that $PA = PB = PD$ and $QA = QB = QC$. 