12 Flanders Mathematics Olympiad 2003

12.1 First round: the problems

For this set of problems there is a working time of max. 3 hours allowed, the scoring system is as follows: a correct answer amounts for 5 points, a wrong answer for 0 points, and a blank answer for 1 point.

1. Knowing that 1 is a solution of the quadratic equation $x^2 + kx + 3 = 0$; determine the other solution.

(A) -4	(B) -3	(C) -1	(D) 2	(E) 3

2. John and Linda are brother and sister. John has twice as many sisters as brothers. Linda has as much sisters as brothers. The number of children in their family equals

(A) 5	(B) 6	(C) 7	(D) 8	(E) 9

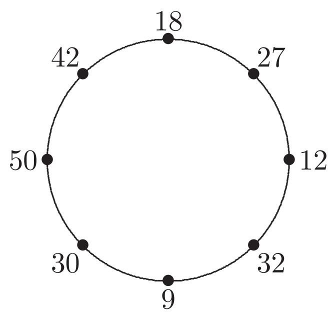
3. A palindrome is a number that is the same if you read it from the end to the front: for example 92129.
The difference between the next year that is a palindrome and the last year that was a palindrome equals
(Note: this question was written in the year 2003.)

(A) 11	(B) 101	(C) 110	(D) 121	(E) 1001

4. The prices in a store are raised bij 10%. Shortly afterwards the prices are lowered by 10%. This means that

(A) the prices stay the same.
(B) the original prices are raised by 1%
(C) the original prices are lowered by 1%.
(D) the original prices are raised by 5%.
(E) the original prices are lowered by 5%.

5. The numbers in the picture are at the vertices of a regular octagon. We connect the numbers that are mutually indivisible by a line segment. The figure formed this way is



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(A) a rhombus (B) a rectangle
(C) a Star of David (D) a rectangular triangle
(E) an isosceles trapezium

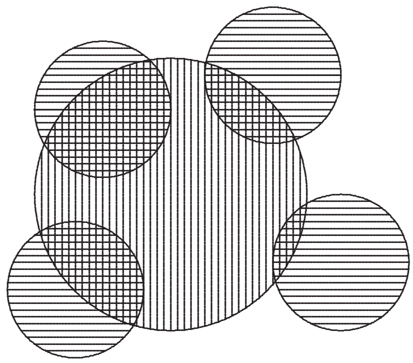
6. The last digit of the sum $6^{2003} + 6^{2002} + \ldots + 6 + 1$ equals

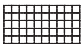
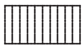
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

7. The numbers 1, 2, 3, 4, 5 are written in a random order to obtain a number consisting of 5 digits. The chance that this number is divisible by 6 is

(A) 16,66... % (B) 33,33... % (C) 40% (D) 50% (E) 60%

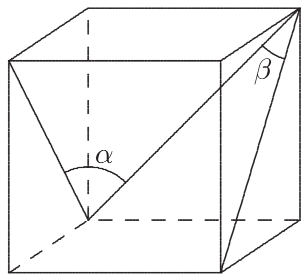
8. The picture shows four equal small circles and one bigger circle with radius double the radius of a smaller circle.
What do you know about the shaded areas using the following notation?



Sum  $=A$, Sum  $=B$,  $=C$

(A) $A > B > C$ (B) $C > A > B$ (C) $A > C > B$
(D) $C > B > A$ (E) none of the previous

9. We connect some of the vertices of a cube as shown in the picture.
What do we know about the angles α and β?

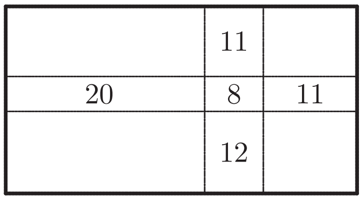


(A) $\alpha > \beta$ (B) $\alpha < \beta$ (C) $\alpha = \beta = 45°$
(D) $\alpha = \beta = 60°$ (E) $\alpha = \beta = 90°$

10. How many integers can be found between 1 and 1000 with the sum of their digits equal to 7?

(A) 40 (B) 36 (C) 32 (D) 28 (E) 24

11. A rectangle is divided into nine smaller rectangles. The perimeter of five rectangles is given (see picture).
Determine the perimeter of the large rectangle.



(A) 32	(B) 46	(C) 48	(D) 67	(E) 69

12. The priest decided to repair the clock of the church himself (to save money). He switched by mistake the hour hand and the minute hand.
How many times between Monday 3 p.m. and Tuesday 3 p.m. does the clock indicate the right time?

(A) 22	(B) 23	(C) 24	(D) 25	(E) 26

13. If $\left(\frac{1}{x}\right)^3 = 0{,}064$ then x equals

(A) 250	(B) 2,5	(C) 0,4	(D) 0,0192	(E) 0,004

14. Consider the number $N = 2^{10} \cdot 10^2$. How many of the following statements are true?

I. N is divisible by 5.
II. N is not divisible by 25.
III. N is divisible by 40.
IV. N is not divisible by 50.

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

15. Consider the following equation in x:

$$1 + a\cos x = (a+1)^2.$$

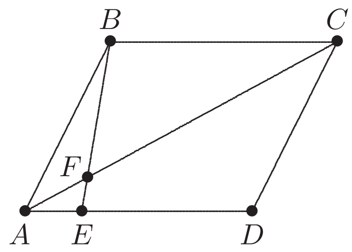
The number of integer values of $a \neq 0$ for which this equation has solutions is

(A) 1	(B) 2	(C) 3	(D) 4	(E) more than 4

16. One connects a vertex B of a parallelogram $ABCD$ with a point E on the side $[AD]$ in such a way that

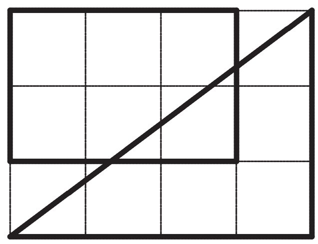
$$|AE| = \frac{1}{4}|AD|.$$

The line segment $[BE]$ intersects the diagonal $[AC]$ in a point F.
The ratio $\dfrac{|AF|}{|AC|}$ is



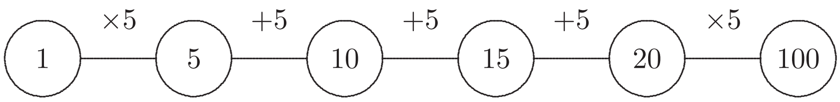
(A) $\frac{1}{8}$	(B) $\frac{1}{6}$	(C) $\frac{1}{5}$	(D) $\frac{1}{4}$	(E) $\frac{1}{2\sqrt{2}}$

17. On a squared piece of paper (with squares of $1\ \text{cm}^2$) a rectangle and a triangle are drawn as shown in the picture. Determine the area of the overlapping area.

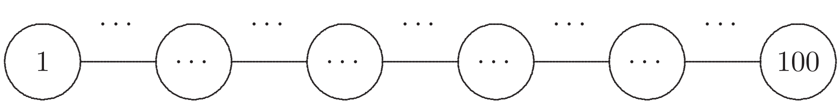


(A) $\frac{7}{8}\ \text{cm}^2$	(B) $1\ \text{cm}^2$	(C) $\frac{9}{8}\ \text{cm}^2$	(D) $\frac{13}{12}\ \text{cm}^2$	(E) $\frac{25}{24}\ \text{cm}^2$

18. Starting with 1 we can obtain the number 100 in 5 steps by either multiplying by 5 or adding 5.

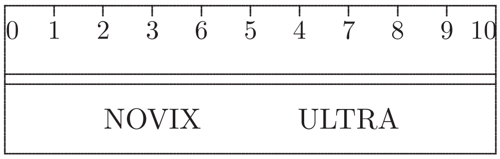


Determine the smallest positive integer x, different from 5, such that we can obtain in 5 steps 100 starting with 1 by either multiplying by x or adding x.



(A) 2	(B) 4	(C) 8	(D) 10	(E) 20

19. James measures the sides of a triangle which have all a different length. Moreover, these lengths are natural numbers. He finds that the perimeter of the triangle equals 15 cm, but this is not correct since the 4 and the 6 are switched on the measuring rule he uses.



Determine the correct perimeter.

(A) 13 cm	(B) 14 cm	(C) 16 cm
(D) 17 cm	(E) not defined from these data	

20. To know the correct date of Easter in the year Y $(1900 < Y < 2099)$, we determine

- the remainder a when dividing Y by 19,
- the remainder b when dividing Y by 4,
- the remainder c when dividing Y by 7,
- the remainder d when dividing $19a + 24$ by 30,
- the remainder e when dividing $2b + 4c + 6d + 5$ by 7.

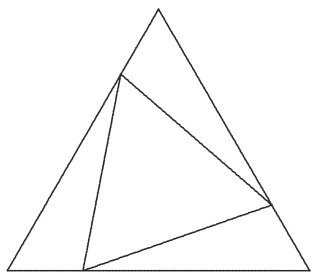
Easter is on $d + e$ days after March 22, except

- if the result is April 26, than Easter is on April 19,
- if $d = 28$ and $e = 6$, than Easter is on April 18.

Determine the date of Easter in the year 2050.

(A) March 26	(B) March 31	(C) April 1	(D) April 10	(E) April 19

21. The sides of an equilateral triangle are divided into pieces that are in the ratio of 4 to 1 in such a way that the dividing points also form an equilateral triangle (see figure). Determine the ratio of the area of the smaller equilateral triangle to the area of the larger equilateral triangle.



(A) $\frac{1}{2}$	(B) $\frac{9}{16}$	(C) $\frac{4}{5}$	(D) $\frac{13}{25}$	(E) $\frac{16}{25}$

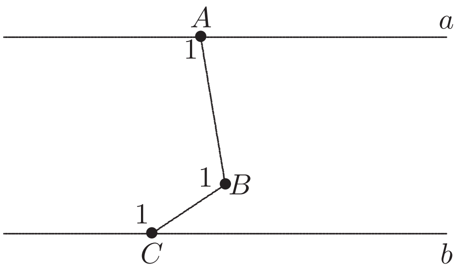
22. A cylinder and a cone have the same altitude and the same volume. The ratio of the radius of the base of the cylinder to the radius of the base of the cone is

(A) 1 to 2	(B) 2 to 3	(C) $\sqrt{3}$ to 3	(D) $\sqrt{2}$ to 2	(E) 1 to 3

23. William exercises during more than seven successive days. The different routes he is following are 5 km, 7 km or 9 km. At the end of the training period he has run 47 km. How many times did he run the route of 9 km?

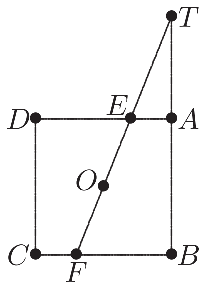
(A) 0 times	(B) 1 time	(C) 2 times
(D) 3 times	(E) more than 3 times	

24. If a is parallel to b and $\hat{A}_1 = 100°$ and $\hat{B}_1 = 120°$ (see picture), then $\hat{C}_1$ equals



(A) 120°	(B) 130°	(C) 140°
(D) 150°	(E) 160°	

25. In the figure a square is cut by a line through the centre O. The angle between that line and one of the sides is $\frac{\pi}{3}$. So:



(A) $\lvert TE\rvert = \lvert DE\rvert$	(B) $\lvert TO\rvert = \lvert AD\rvert$	(C) $\lvert TA\rvert = 2\lvert EA\rvert$
(D) $\lvert OA\rvert = 2\lvert EA\rvert$	(E) $\lvert FB\rvert = \lvert TO\rvert$	

26. Which step is wrong in the following reasoning?

$$\begin{array}{rcl}
1 < 3 & \overset{(1)}{\Rightarrow} & \frac{1}{27} < \frac{3}{27} \\
\frac{1}{27} < \frac{3}{27} & \overset{(2)}{\Rightarrow} & \left(\frac{1}{3}\right)^3 < \left(\frac{1}{3}\right)^2 \\
\left(\frac{1}{3}\right)^3 < \left(\frac{1}{3}\right)^2 & \overset{(3)}{\Rightarrow} & \log\left(\frac{1}{3}\right)^3 < \log\left(\frac{1}{3}\right)^2 \\
\log\left(\frac{1}{3}\right)^3 < \log\left(\frac{1}{3}\right)^2 & \overset{(4)}{\Rightarrow} & 3\log\left(\frac{1}{3}\right) < 2\log\left(\frac{1}{3}\right) \\
3\log\left(\frac{1}{3}\right) < 2\log\left(\frac{1}{3}\right) & \overset{(5)}{\Rightarrow} & 3 < 2
\end{array}$$

(A) (1)	(B) (2)	(C) (3)	(D) (4)	(E) (5)

27. Consider the three functions

$$f(x) = |x| + |2x|, \quad g(x) = |3x|, \quad h(x) = |4x| - |x|$$

So:

(A) $f = g = h$	(B) $f = g < h$	(C) $h < f = g$
(D) $h < g < f$	(E) $f < g < h$	

28. Given a non-constant geometric sequence a_1, a_2, a_3, $\ldots$ where

$$s = a_1 + a_2 + a_3 + \cdots + a_7 + a_8 + a_9 + a_{10} = 3(a_1 + a_3 + a_5 + a_7 + a_9).$$

Then

(A) $\frac{s}{a_1} \in [250, 300]$	(B) $\frac{s}{a_1} \in [500, 550]$	(C) $\frac{s}{a_1} \in [1000, 1050]$
(D) $\frac{s}{a_1} \in [2000, 2050]$	(E) $\frac{s}{a_1}$ not defined from these data	

29. You have four sticks. The length of the second stick is 80% of the length of the first stick, the length of the third stick is 40% of the length of the first stick and the length of the fourth stick is 20% of the length of the first stick.
How many non-congruent triangles can be formed using three of these four sticks?

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

30. When dividing the polynomial $x^3 + x^5 + x^7 + x^{11} + x^{13} + x^{17} + x^{19} + x^{23} + x^{29}$ by the polynomial $x^2 - 1$ there is a remainder. The numerical value of that remainder for $x = 2$ is

(A) 2	(B) 3	(C) 9	(D) 18	(E) 27

12.2 First round: the answer patterns

Flanders Mathematics Olympiad 2003 – First round Answer patterns (all participants)									
Problem	Correct	A	B	C	D	E	Correct	Wrong	Blank
1	E	7.77	5.97	4.02	1.57	74.22	74.22	19.34	6.44
2	C	6.86	5.85	81.54	2.21	0.86	81.54	15.78	2.68
3	C	5.98	0.99	82.16	2.59	4.92	82.16	14.49	3.35
4	C	4.47	3.25	90.07	0.53	0.91	90.07	9.18	0.74
5	E	0.90	1.10	5.10	3.01	82.05	82.05	10.15	7.80
6	E	7.05	5.28	19.81	3.42	35.31	35.31	35.60	29.09
7	C	27.11	9.92	25.78	4.60	2.40	25.78	44.04	30.18
8	E	3.77	29.74	23.95	2.59	25.62	25.62	60.07	14.31
9	D	4.46	0.79	9.40	49.64	30.82	49.64	45.53	4.83
10	B	5.67	64.37	5.53	7.32	4.17	64.37	22.69	12.95
11	B	2.13	51.14	9.11	4.02	2.54	51.14	17.84	31.02
12	A	12.56	14.08	42.51	6.40	3.79	12.56	66.82	20.62
13	B	10.16	59.87	16.48	1.94	4.82	59.87	33.44	6.69
14	C	1.87	7.38	71.15	8.08	9.54	71.15	26.92	1.93
15	C	13.98	15.54	26.09	2.92	10.11	26.09	42.55	31.36
16	C	2.96	13.21	37.68	11.55	3.52	37.68	31.25	31.07
17	E	1.50	10.48	10.51	19.46	36.62	36.62	41.95	21.42
18	B	0.88	45.73	1.95	0.86	42.72	45.73	46.42	7.85
19	D	8.83	0.84	0.64	7.60	68.87	7.60	79.20	13.20
20	D	2.91	2.63	3.69	55.03	9.12	55.03	18.38	26.58
21	D	12.39	13.23	5.36	13.89	9.07	13.89	40.06	46.06
22	C	14.81	4.73	20.87	6.38	18.82	20.87	44.74	34.39
23	A	42.00	11.13	18.42	13.11	2.14	42.00	44.85	13.15
24	C	4.95	3.13	74.30	4.58	5.96	74.30	18.65	7.04
25	B	16.01	10.65	13.23	9.32	1.54	10.65	40.12	49.23
26	E	2.88	16.97	30.16	11.93	22.60	22.60	61.97	15.43
27	A	85.95	1.90	1.21	0.95	1.27	85.95	5.38	8.68
28	C	1.44	2.81	4.67	1.72	27.13	4.67	33.12	62.21
29	B	8.70	21.13	12.72	11.16	22.60	21.13	55.19	23.69
30	D	32.89	6.50	4.50	7.58	1.99	7.58	45.89	46.53

12.3 Second round: the problems

For this set of problems there is a working time of max. 2 hours allowed, and the scoring system is as follows: a correct answer amounts for 5 points, a wrong answer for 0 points, and a blank answer for 1 point.

1. The number of perfect squares between 5^4 and 4^5 is

(A) 2	(B) 3	(C) 4	(D) 5	(E) more than 5

2. A rectangle has a side of length a and a diagonal of lenght $2a$. Determine its area.

(A) a^2	(B) $a^2\sqrt{a}$	(C) $a^2\sqrt{3}$	(D) $2a^2$	(E) $a^2\sqrt{5}$

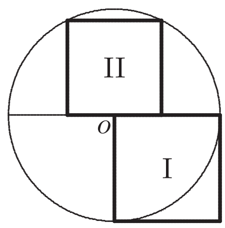
3. Laurien, Lennert and Lisanne went bird-watching. Each of them saw one bird that the others didn't see. Each of them didn't see one bird the two others did see. One bird was seen by the three of them. Two of the birds Laurien saw were yellow. Three of the birds Lennert saw were yellow. Four of the birds Lisanne saw were yellow.
How many yellow birds were watched by them?

(A) 5	(B) 6	(C) 7	(D) 8	(E) 9

4. The number of integer solutions of the inequality $\sqrt{x+2} > x$ equals

(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

5. Square I has two sides that coincide with radii of a circle and square II has two vertices on the same circle and two vertices on the diameter of that circle (see picture).
The ratio of the area of square I to the area of square II equals



(A) 1	(B) 1,2	(C) 1,25	(D) $\sqrt{2}$	(E) 1,5

6. A regular polygon with 100 vertices has n diagonals.
How many diagonals more are there in a regular polygon with 101 vertices?

(A) 98	(B) 99	(C) 100	(D) 101	(E) 102

7. An equilateral triangle with side a is rotated about an axis through one of its vertices and parallel to the opposite side.
Determine the volume of the described body.

(A) $\pi a^3 \frac{\sqrt{3}}{12}$	(B) $\frac{1}{4}\pi a^3$	(C) $\frac{1}{3}\pi a^3$	(D) $\frac{1}{2}\pi a^3$	(E) πa^3

8. One night Frederik drives home from work (100 km) with an average speed of 120 km/h. At the end of the trip, he goes to the gas station just around the corner to refuel. This takes 4 minutes. His wife Veerle drives home from work (also 100 km) with an average speed of 120 km/h during the first 80 km and with an average speed of 140 km/h during the last 20 km. She als goes to the gas station to refuel. This costs her 6 minutes.
If Frederik and Veerle leave at the same time, Veerle arrives home

(A) 1 minute earlier than Frederik.
(B) 2 minutes earlier than Frederik.
(C) 3 minutes earlier than Frederik.
(D) 4 minutes earlier than Frederik
(E) later than Frederik.

9. A line d with slope 2 intersects with the y-axis in $(0, 1)$.
We obtain a second line by reflecting d with respect to the x-axis.
We obtain a third line by reflecting d with respect to the y-axis.
We obtain a fourth line by reflecting d with respect to the first bisector.

Which one of the following equations is NOT the equation of one of these four lines?

(A) $y = 2x + 1$	(B) $y = -2x + 1$	(C) $y = 2x - 1$
(D) $y = -2x - 1$	(E) $y = \frac{1}{2}(x - 1)$	

10. How many statements are correct about the real solutions of $x^7 + x + 1$?

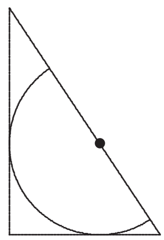
I. All of the solutions are positive.
II. At least one of the solutions is positive.
III. All of the solutions are negative.
IV. All of the solutions are between -1 and 0.
V. There are exactly four different real solutions.

(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

11. In a rectangular triangle the bisector of a sharp angle α divides the opposite side into two parts. The ratio of the length of the largest part to the length of the smallest part is as 1 to

(A) $\cos\alpha$	(B) $\sin\alpha$	(C) $\tan\frac{\alpha}{2}$	(D) $\sin\frac{\alpha}{2}$	(E) $\cos\frac{\alpha}{2}$

12. In a rectangular triangle with rectangular sides of length 4 and 6, we construct a half-circle with center on the hypotenuse and being tangent to the rectangular sides. Determine the radius of this circle.



(A) 2	(B) 2,4	(C) 2,5	(D) 3	(E) $\dfrac{2}{3\sqrt{13}}$

13. The graph of the function

$$f(x) = \frac{4x}{x - |x|}$$

in an orthogonal coordinate system consists of

(A) two half-lines, not parallel.
(B) two parallel half-lines (not on the same line).
(C) one half-line parallel to the x-axis.
(D) one half-line not parallel to one of the two axes.
(E) one line with a gap of one point.

14. If we solve the following system of equations in variables x and y in $\mathbb{R}$

$$\begin{cases} x^3y^4 = a \\ x^5y^6 = b \end{cases} \quad a \cdot b > 0$$

we find that $x =$

(A) $\dfrac{b^2}{a^3}$	(B) $\dfrac{2b}{3a}$	(C) $\sqrt{\dfrac{a^5}{b^3}}$	(D) $b^2 - a^2$	(E) $2b - 3a$

15. The function $f : \mathbb{R} \to \mathbb{R} : x \mapsto |\cos x| + \cos|x|$

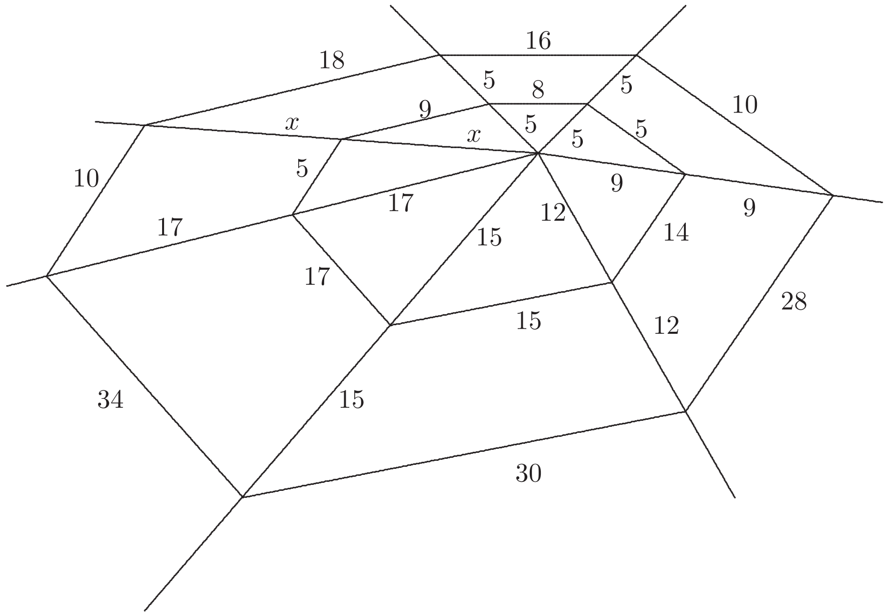
(A) is periodical with period $\frac{\pi}{4}$.
(B) is periodical with period $\frac{\pi}{2}$.
(C) is periodical with period π.
(D) is periodical with period 2π.
(E) is not periodical.

16.

$$2003 - 2001 + 1999 - \ldots - 5 + 3 - 1 =$$

(A) 2	(B) 1000	(C) 1002	(D) 1020	(E) 2004

17. A mathematically skilled spider spun a web where the lengths of all strings are integer numbers (as shown in the picture).



Determine x.

(A) 11	(B) 13	(C) 15	(D) 17	(E) 19

18. What is the last digit of the product $1 \cdot 3 \cdot 5 \cdot 7 \ldots 2001 \cdot 2003$?

(A) 1	(B) 3	(C) 5	(D) 7	(E) 9

19. If you know that

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\ldots=\frac{\pi^2}{6}$$

then determine

$$1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\frac{1}{81}+\ldots$$

(A) $\frac{\pi^2}{7}$	(B) $\frac{\pi^2}{8}$	(C) $\frac{\pi^2}{9}$	(D) $\frac{\pi^2}{10}$	(E) $\frac{\pi^2}{12}$

20. The remainder of the division of $(1!+2!+3!+4!+5!+6!+7!+8!+9!)^2$ by 5 equals ($n! = 1\cdot 2\cdot 3\cdot\ldots\cdot(n-2)\cdot(n-1)\cdot n$; e.g. $6! = 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6$)

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

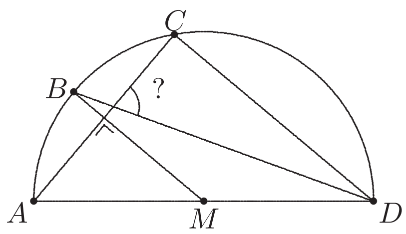
21. On the planet Quaternion one calculates with our real numbers and the well known multiplication, but also with three other symbols i, j and k which are multiplied as follows:

$$i\cdot i=-1;\ j\cdot j=-1;\ k\cdot k=-1;\ i\cdot j=k;\ j\cdot k=i;\ k\cdot i=j.$$

It is known, moreover, that the multiplication on Quaternion is associative but not commutative. Thus $k\cdot j\cdot i$ equals

(A) 1	(B) -1	(C) i	(D) j	(E) k

22. In the figure AD is the diameter of a circle with center M. The two points B and C are on the circle in such a way that $AC \perp BM$ en $\hat{A} = 50°$.
Determine the angle between the lines AC and BD (shown on the figure).



(A) 50°	(B) 60°	(C) 65°	(D) 70°	(E) 75°

23. The number of different real solutions of the equation

$$\sqrt[3]{x}+\sqrt[3]{7-x}=3$$

equals

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

24. The sum of the first $5n$ strictly positive integer numbers is 1210 less than the sum of the first $7n$ strictly positive integer numbers. Determine n.

(A) 6	(B) 8	(C) 9	(D) 10	(E) 12

25. If we factorize the polynomial $x^8 - 1$ as a product of as many as possible real polynomials, how many factors do we obtain?

(A) 2	(B) 4	(C) 5	(D) 6	(E) 8

26. The point $A(4, 0)$ and $B(12, 0)$ are given fixed points in an orthonormal coordinate system. A triangle is formed with these two points and the point $P(k, k)$.
Determine the value of k so that the perimeter of the triangle is as small as possible.

(A) 3	(B) 4	(C) 5	(D) 6	(E) 8

27. If one chooses two arbitrary, different numbers out of the set, $\{1, 2, 3, \ldots, n-1, n\}$, the probability that the numbers are successive natural numbers is 20%.
Determine n.

(A) 5	(B) 6	(C) 10
(D) 11	(E) none of the previous	

28. Suppose that $f(x) = \sqrt{x - 2}$.
Which one of the following expressions represents the smallest real number?

(A) 123	(B) $f(123)$	(C) $f(f(123))$
(D) $f(f(f(123)))$	(E) $f(f(f(f(123))))$	

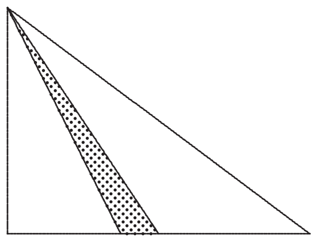
29. The sum of all numbers α between 0 and 2π that satisfy

$$\tan^2 \alpha - 2003 \tan \alpha + 1 = 0$$

equals

(A) $\frac{\pi}{2}$	(B) π	(C) 2π	(D) $\frac{5\pi}{2}$	(E) 3π

30. Consider a rectangular triangle with rectangular sides of length 30 and 40. If we draw the median and the bisector from the largest acute angle, we obtain a new triangle. Determine the area of that triangle.



(A) 50	(B) 70	(C) 75	(D) 80	(E) 150

12.4 Second round: the answer patterns

Flanders Mathematics Olympiad 2003 – Second round Answer patterns (all participants)									
Vraag	Juist	A	B	C	D	E	Juist	Fout	Blanco
1	E	4.11	2.21	2.11	3.69	73.66	73.66	12.12	14.23
2	C	2.32	0.32	93.36	1.48	1.58	93.36	5.69	0.95
3	A	66.39	7.27	9.27	1.69	2.95	66.39	21.18	12.43
4	D	9.48	29.72	6.95	49.63	0.74	49.63	46.89	3.48
5	C	2.85	8.85	45.84	7.38	3.37	45.84	22.44	31.72
6	B	21.18	55.95	7.48	1.58	1.79	55.95	32.14	11.91
7	D	12.43	12.12	8.01	26.13	2.42	26.13	34.98	38.88
8	E	5.16	0.95	0.84	0.21	90.20	90.20	7.17	2.63
9	C	1.26	7.17	69.86	2.42	13.59	69.86	24.55	5.58
10	B	6.32	52.90	8.01	1.05	0.21	52.90	15.60	31.51
11	A	12.33	4.32	12.54	10.75	6.95	12.33	34.56	53.11
12	B	4.64	52.90	4.95	3.16	5.27	52.90	18.12	28.98
13	C	1.16	2.11	74.08	9.91	2.42	74.08	15.60	10.33
14	A	50.68	1.79	3.90	1.48	0.84	50.68	8.01	41.31
15	D	1.48	6.32	19.39	39.62	6.22	39.62	33.40	26.98
16	C	4.11	8.43	68.18	0.84	11.38	68.18	24.76	7.06
17	B	6.11	23.29	3.69	2.74	2.11	23.29	14.65	62.07
18	C	0.42	2.11	81.77	0.95	0.53	81.77	4.00	14.23
19	B	4.85	12.43	7.48	4.95	10.96	12.43	28.24	59.33
20	E	9.17	3.69	3.48	29.19	34.14	34.14	45.73	20.13
21	A	47.42	30.98	0.53	1.48	0.74	47.42	33.72	18.86
22	D	1.37	2.21	2.21	69.65	3.58	69.65	9.38	20.97
23	C	16.75	10.54	7.17	3.58	1.05	7.17	31.93	60.91
24	D	1.58	2.95	5.80	33.93	4.11	33.93	14.44	51.63
25	C	6.64	65.23	5.90	2.32	6.64	5.90	80.82	13.28
26	A	30.66	27.29	4.64	3.37	7.38	30.66	42.68	26.66
27	C	7.59	13.70	24.76	14.44	16.54	24.76	52.27	22.97
28	D	0.63	1.58	4.85	68.91	11.80	68.91	18.86	12.22
29	E	5.37	5.69	6.95	5.37	4.74	4.74	23.39	71.87
30	C	4.00	4.74	25.50	5.48	6.74	25.50	20.97	53.53

12.5 The final round

To solve these 4 problems, a working time of 3 hours is allowed.

1. **A soccer game.**
   Three friends, Ronny, Steven and Tom, are playing soccer in the afternoon. As doing this with only three players is rather difficult, they agree on the following: two boys are field-players and they try to score. The third boy is the goalkeeper.

   If one of the field-players scores, an new game starts: the goalkeeper becomes field-player and the one who scored becomes the keeper.

   During the afternoon, Ronny is 12 times a field-player and Steven is 21 times field-player. Tom is goalkeeper during 8 games.

   Who scores the sixth goal?

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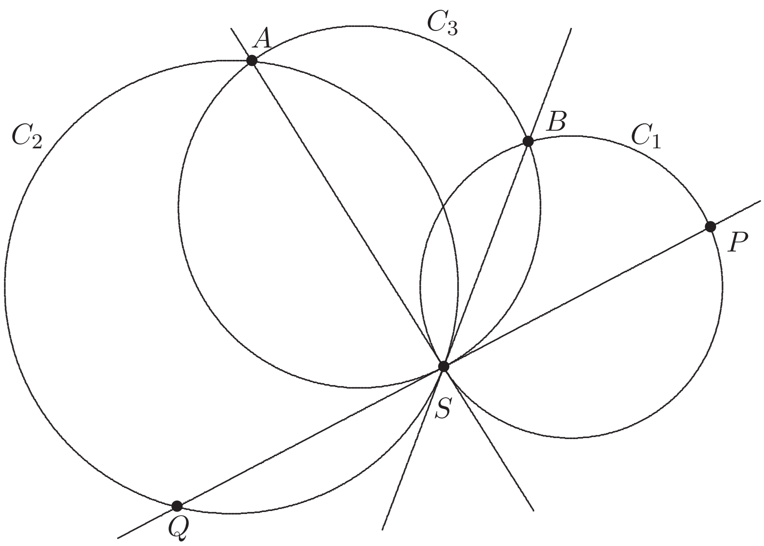
2. Two circles C_1 and C_2 are intersecting in the point S. The tangent line in S to C_1 intersects C_2 in $A \neq S$ and the tangent line in S to C_2 intersects C_1 in $B \neq S$. A third circle C_3 goes through A, B and S. The tangent line in S to C_3 intersects C_1 in $P \neq S$ and C_2 in $Q \neq S$.



   Proof that $|PS| = |QS|$.

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3. A number consists of three different digits. The sum of the other five numbers that can be formed with those three digits is 2003.
   Determine that number.

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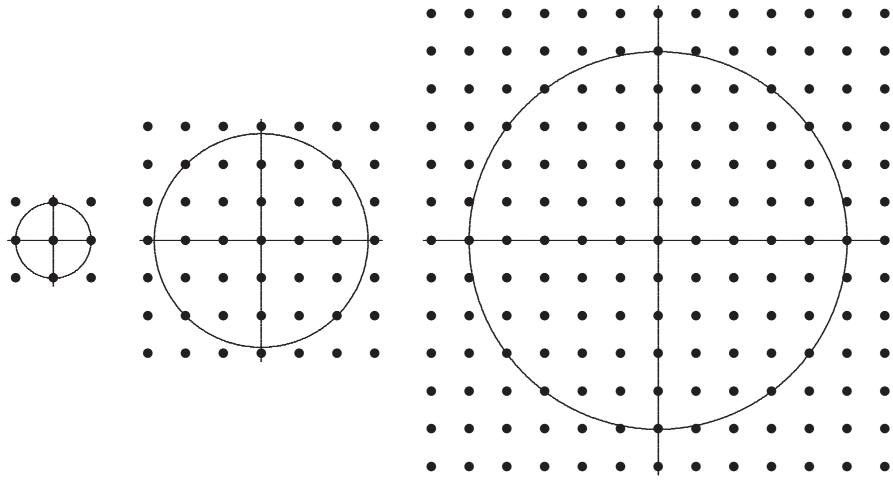
4. Consider, in the plane, the grid of all the points with integer coordinates. If one chooses the number R well, the circle with centre $(0,0)$ of radius R passes through some grid points. For example, the circle of radius 1 with centre $(0,0)$ passes through 4 grid points. The circle of radius $2\sqrt{2}$ passes through 4 grid points and the circle of radius 5 passes through 12 grid points.



Proof that for each $n \in \mathbb{N}$ there exists a number R so that the circle of radius R with centre $(0,0)$ passes through at least n grid points.