Large maximal partial spreads of the Hermitian variety

\[ H(5, q^2) \]

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We consider the Hermitian variety in 5-dimensions, denoted by \( H(5, q^2) \). This is an example of a finite classical polar space of rank 3. The Hermitian variety \( H(5, q^2) \) contains points, lines and planes of the ambient projective space \( PG(5, q^2) \). The planes contained in \( H(5, q^2) \) are called \textit{generators}.

A \textit{spread} of \( H(5, q^2) \) is a set \( S \) of generators such that every point of \( H(5, q^2) \) is contained in exactly one element of \( S \). A spread contains exactly \( q^5 + 1 \) elements. A \textit{partial spread} of \( H(5, q^2) \) is a set \( S \) of generators such that every point of \( H(5, q^2) \) is contained in at most one element of \( S \). A partial spread is called \textit{maximal} if no generator of \( H(5, q^2) \setminus S \) can be added to \( S \).

Since spreads of \( H(5, q^2) \) does not exist by a result of J. A. Thas ([3]), the natural question is how many elements the largest maximal partial spread contains.

Using counting arguments and the particular geometrical structure, we can improve the known upper bounds ([3] and [2]) and show that a maximal partial spread contains at most \( q^3 + 1 \) elements. Furthermore, from [1], we know that any spread of the symplectic polar space \( W(5, q) \) embedded in \( H(5, q^2) \), constitutes a maximal partial spread of \( H(5, q^2) \), of size \( q^3 + 1 \). Hence, the new upper bound is sharp.

References

