MEET THE EXPERT: ANDREAS DEFANT ON DIRICHLET SERIES AND HARALD BOHR'S LEGACY

27/01–30/01, UGENT

In the week of January 27, Prof. Andreas Defant from the University of Oldenburg will visit Ghent University. He will give four one hour lectures, from Monday 27/01 until Thursday 30/01. The first lecture is intended for a broad mathematical audience and is about the legacy of Harald Bohr in contemporary analysis. The next three lectures form a mini series on summation of Dirichlet series, and are of a more specialized nature. The lectures will be followed by a sandwich lunch.

Participation is free (including lunch), but we ask you to register by 17/01 by sending an email to fabrouck.broucke@ugent.be, indicating which lectures you will attend.

Lecture 1: The legacy of Harald Bohr in contemporary analysis
Time and place: Monday 27/01, 10:30, Leslokaal 3.1, S8, campus Sterre, Krijgslaan 281, Gent
Abstract: Over 110 years ago, Harald Bohr began systematically studying Dirichlet series with the primary goal of resolving the Riemann hypothesis. Although he did not succeed in this endeavor, his work paved the way for a resurgence of interest in the field. In recent years, this revival has led to a dynamic interplay between classical ideas and modern developments in functional analysis, harmonic analysis, infinite-dimensional holomorphy, probability theory, and analytic number theory.

This talk will offer a brief overview of key milestones in this evolving area of research, from Bohr's famous absolute convergence problem to contemporary topics like quantum information theory.

Lecture 2–4: Summation of Dirichlet series: past and present
Time and place: Tuesday 28/01–Thursday 30/01, 10:30, Leslokaal 3.1, S8, campus Sterre, Krijgslaan 281, Gent
Abstract: We plan three lectures on Bohr's theorem: Let $\sum a_n n^{-s}$ be an ordinary Dirichlet series which converges pointwise on some half-plane, where its limit function extends to the entire right half-plane $\mathrm{Re}(s) > 0$ as a bounded holomorphic function. Then $\sum a_n n^{-s}$ also converges uniformly on any smaller half-plane $\mathrm{Re}(s) > \epsilon$, for $\epsilon > 0$.

Much of the abstract theory of ordinary Dirichlet series relies on Bohr's theorem. However, a similar phenomenon does not hold for λ-Dirichlet series of the form $\sum a_n e^{-\lambda_n s}$, where $\lambda = (\lambda_n)$ represents an arbitrary frequency, i.e., $0 \leq \lambda_n \to \infty$ strictly. So moving from ordinary Dirichlet series $\sum a_n n^{-s}$ to λ-Dirichlet series $\sum a_n e^{-\lambda_n s}$ introduces significant complications. We define a frequency λ as satisfying Bohr's theorem if the result holds for all λ-Dirichlet series (instead of ordinary Dirichlet series).

In the first two lectures, we will explore the following meta-theorem: Given a frequency λ, any reasonable structure theory for λ-Dirichlet series is possible if and only if λ satisfies Bohr's theorem. In the third lecture, we will examine how Bohr's theorem can be scaled

to handle unbounded λ-Dirichlet series, i.e., Dirichlet series whose limit functions have an unbounded extension to the right half-plane $\mathrm{Re}(s) > 0$.

All results are primarily based on the so-called Riesz summation of Dirichlet series, a technique dating back to Harald Bohr and Marcel Riesz at the beginning of the last century. We aim to revive this method within the framework of modern Fourier analysis.

Most results are joint work with Ingo Schoolmann.

This initiative is supported by Flanders: State of the Art via the doctoral schools program.