The (extended) rank weight enumerator and q-matroids

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(Joint work with Ruud Pellikaan)

Let C be an $\mathbb{F}_{q^m}$-linear code of length n. With the element $\mathbf{c} = (c_1, \dots, c_n)$ of C an $m \times n$ matrix $m(\mathbf{c})$ is associated where the j-th column of $m(\mathbf{c})$ consists of the coordinates of c_j with respect to a fixed basis of $\mathbb{F}_{q^m}$ over $\mathbb{F}_q$. The rank weight $\mathrm{wt}_R(\mathbf{c}) = \mathrm{rk}(\mathbf{c})$ of $\mathbf{c}$ is by definition the rank of the matrix $m(\mathbf{c})$. The *rank weight enumerator* is given by

$$W_C^R(X, Y) = \sum_{w=0}^{n} A_w^R X^{n-w} Y^w,$$

where $A_w^R = |\{\mathbf{c} \in C : \mathrm{rk}(\mathbf{c}) = w\}|$. See [1].

The purpose of this talk is to investigate the rank weight enumerator of a code over $\mathbb{F}_{q^m}$ and its relation with the Tutte polynomial of the *q-matroid* of the code. This can be viewed as the q-analogon of the Hamming weight enumerator of a code over $\mathbb{F}_q$ and its relation with the Tutte polynomial of the matroid of the code, see [2].

The q-matroid will be defined on a vector space $\mathbb{F}_q^n$ instead of a finite set of n elements. We will furthermore define q-rank and the notion of q-independent subspaces of the q-matroid. Finally, we will show the relation between the Tutte polynomial of a q-matroid and the rank weight enumerator of its corresponding code.

References

[1] E. M. Gabidulin (1985). *Theory of codes with maximal rank distance.* Problems of Information Transmission 21(1), 1–12.

[2] R. Jurrius and R. Pellikaan (2013). *Codes, arrangements and matroids.* In: Martinez-Moro, E. (Ed.), Algebraic geometry modeling in information theory, (pp. 219–325). London: World Scientific.

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